HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate

Trial Examination Term 3 2024

STUDENT NUMBER: ______________________________

TEACHER NAME: Ms Guan, Ms Murray

STUDENT NAME: ______________________________ **(circle one)** Mr Payne, Mrs Sztajer

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–10)

- Attempt Questions 11–14
- Start each question in a new writing booklet
- Write your student number on every writing booklet
- Allow about 1 hour and 45 minutes for this section

Question	*1-10*	*11*	*12*	*13*	*14*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

Outcomes assessed: ME 12-1, 12-2, 12-3, 12-4, 12-7

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

Question 1

What is the size of the angle between the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j}$?

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Question 2

Which of the following integrals is equivalent to $\int \cos^2 4x \, dx$?

(A) $\int \frac{1-\cos 8x}{2} \, dx$

(B) $\int \frac{1+\cos 8x}{2} \, dx$

(C) $\int \frac{1-\cos 4x}{2} \, dx$

(D) $\int \frac{1+\cos 4x}{2} \, dx$

Question 3

What is the remainder when $P(x)=x^3-3x^2+2x+3$ is divided by $(x+1)$?

(A) –3

(B) –2

(C) 2

(D) 3

Question 4

Consider the differential equation $\frac{dy}{dx}=\frac{1}{xy}$.
Which of the following equations best represents this relationship between x and y ?

(A) $y=\frac{1}{x}+c$

(B) $y=\ln|x|+c$

(C) $y^2=\ln|x|+c$

(D) $y^2=2\ln|x|+c$

Question 5

For the two non-zero vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ it is known that $\overrightarrow{OA}\bullet\overrightarrow{OB}=0$.

Which of the following MUST be true?

(A) Either $\overrightarrow{OA}=0$ or $\overrightarrow{OB}=0$

(B) $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are parallel

(C) $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular

(D) O, A and B are collinear

Question 6

What is the constant term in the binomial expansion $\left(3x-\frac{2}{x^2}\right)^9$?

(A) $\binom{9}{3}3^3.2^6$

(B) $\binom{9}{6}3^6.2^3$

(C) $-\binom{9}{3}3^3.2^6$

(D) $-\binom{9}{6}3^6.2^3$

Question 7

Which of the following is equivalent to $\sin x+\sqrt{3}\cos x$ expressed in the form $A\cos(x+\theta)$?

(A) $2\cos\left(x-\frac{\pi}{6}\right)$

(B) $2\cos\left(x+\frac{\pi}{6}\right)$

(C) $4\cos\left(x-\frac{\pi}{6}\right)$

(D) $4\cos\left(x+\frac{\pi}{6}\right)$

Question 8

Six adults and four children need to be seated at a circular table. How many arrangements exist if two particular children must be separated?

(A) 10 080

(B) 17 280

(C) 282 240

(D) 362 880

Question 9

Which of the following pairs of parametric equations are **NOT** equivalent to $y = \sqrt{x+1}$?

(A) $x = t^2 - 1,\ y = t\ \ for\ t \geq 0$

(B) $x = t,\ y = \sqrt{t+1}\ \ for\ t \geq 1$

(C) $x = t - 1,\ y = \sqrt{t}\ \ for\ t \geq 0$

(D) $x = t - 2,\ y = \sqrt{t-1}\ \ for\ t \geq 1$

Question 10

What is the domain of the function $y = \sin(\arcsin x)$?

(A) $-1 \leq x \leq 1$

(B) $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

(C) $0 \leq x \leq \pi$

(D) All real x

Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section
Instructions

- Answer the questions in the appropriate writing booklet.
- In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) Find $\left(2\underset{\sim}{i}+3\underset{\sim}{j}\right)-\left(\underset{\sim}{i}-2\underset{\sim}{j}\right)$. **1**

(b) Use the $t=\tan\dfrac{x}{2}$ to show that $\sin x-\tan\dfrac{x}{2}=\tan\dfrac{x}{2}\cos x$. **2**

(c) Use the substitution $u=x+2$ to find $\int x\sqrt{x+2}\,dx$. **3**

(d) Solve the inequality $8-|2x-1|\le 5$. **2**

(e) A cylindrical tank of radius 2 m is being filled with water so that the volume is increasing at a constant rate of 3 m^3/min.

Find the rate of increase of the depth of the water in the tank. **2**

(f) Evaluate $\int_0^1 \dfrac{dx}{x^2+3}$ **2**

(g) Two swimmers want to swim from a point A on one island to point B on another island where B is due east of A. The ocean has a current of 1.2 ms^{-1} in the direction of 135°T.
Swimmer X swims at 3 ms^{-1} in the direction of 063.43°T in order to reach point B.
Swimmer Y swims at 2.4 ms^{-1}.
On which bearing does swimmer Y need to swim in order to reach point B? **3**

End of Question 11

Question 12 (15 marks) Start a new writing booklet and use the diagrams provided.

(a) Consider function $f(x) = 2^x - 3$,

(i) On the number plane provided on the separate sheet, sketch the graph of $y = f(x)$ showing the x- and y- intercept in exact values. **2**

(ii) Find the equation of the inverse function $f^{-1}(x)$ as a natural logarithm function and state its domain and range. **3**

(iii) On the same number plane for part (i), sketch the graph of $y = f^{-1}(x)$. **1**

(b) The graph of a particular solution to a differential equation passes through the point $(0,1)$. On the slope field provided on the separate sheet, sketch the graph of this particular solution. **1**

(c) A hard-boiled egg at $98°C$ is put in a room at $18°C$. After 5 minutes, the egg's temperature is $58°C$. The rate of cooling of the egg is proportional to the excess of the temperature T of the egg over the temperature S of the room, i.e. $\frac{dT}{dt} = k(T - S)$ where k is a constant and t is time in minutes.

(i) Show that $k = \frac{-\ln 2}{5}$. **2**

(ii) When will the temperature of the egg drop to $20°C$? Correct answer to the nearest minute. **2**

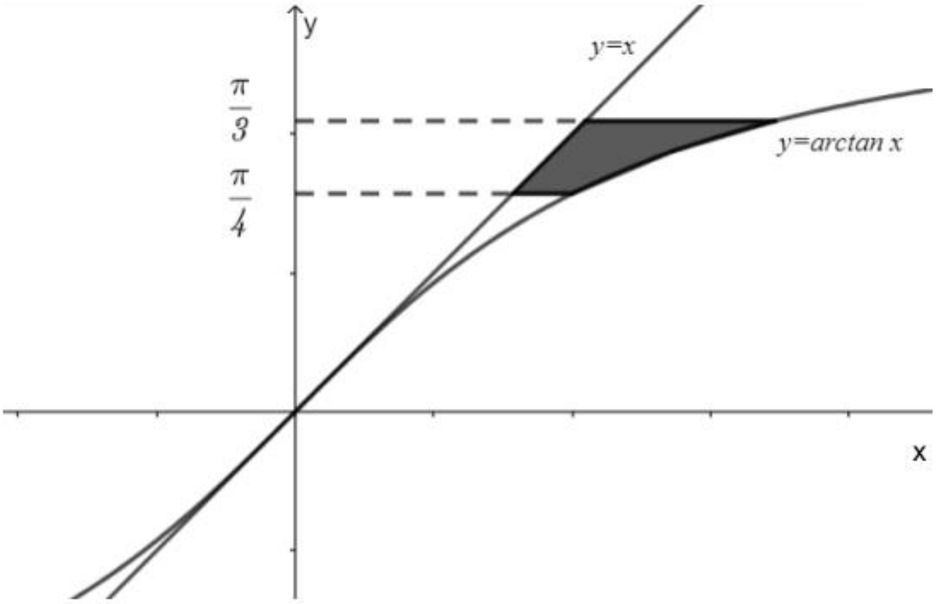
(d) (i) Show that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta}{\cos\theta + \cos 3\theta + \cos 5\theta} = \tan 3\theta$ **2**

(ii) Hence solve $\frac{\sin\theta + \sin 3\theta + \sin 5\theta}{\cos\theta + \cos 3\theta + \cos 5\theta} = 1$, where $0 \le \theta \le 2\pi$. **2**

End of Question 12

Question 13 (15 Marks) Start a new writing booklet.

(a) Find the volume formed when the region between the curve $y = \tan^{-1} x$ and the lines $y = x$, $y = \frac{\pi}{3}$ and $y = \frac{\pi}{4}$, is rotated around the y axis, correct to 2 decimal places. **4**



Not To Scale

(b) Prove by mathematical induction that, for all integers $n \geq 1$, **3**

$$\frac{2}{1\times 3}+\frac{2}{2\times 4}+\frac{2}{3\times 5}+\ldots+\frac{2}{n(n+2)}=\frac{3}{2}-\frac{2n+3}{(n+1)(n+2)}$$

Question 13 continues on the next page

Question 13 continued

(c) At a carnival, an air cannon fires a cylinder containing a t-shirt at time $t = 0$ seconds from an origin O at ground level across a level field. The position vector is

$$\underset{\sim}{r}(t) = 15\sqrt{3}t\underset{\sim}{i} + (15t - 4.9t^2)\underset{\sim}{j},$$

Where $\underset{\sim}{i}$ is a unit vector in the forward direction, $\underset{\sim}{j}$ is a unit vector vertically up and displacement components are measured in metres.

(i) Find the initial velocity of the t-shirt and the initial angle, in degrees, of its trajectory to the horizontal. **2**

(ii) Find the maximum height reached by the t-shirt, giving your answer in metres to two decimal places. **2**

(iii) Find the time of the flight of the t-shirt. Give your answer in seconds, correct to three decimal places. **1**

(iv) Find the range of the t-shirt in metres, correct to 1 decimal place. **1**

(v) A person in the crowd, more than 40 m from *O*, catches the t-shirt at a height of 2 m above the ground. How far horizontally from O, is this person when the t-shirt is caught? Give your answer to one decimal place **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet.

(a) The roots of $x^3 + 3x^2 - 4 = 0$ are α, β and γ. **2**

What is the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\beta\gamma} + \dfrac{1}{\alpha\gamma}$?

(b) The population $P(t)$ of bacteria in a petri dish is modelled by the logistic differential equation $\dfrac{dP}{dt} = \dfrac{P}{6}\left(1 - \dfrac{P}{8000}\right)$ where $P(0) = P_0$ and t is the time in hours.

(i) If the initial population P_0 is 1000 bacteria, show that $P(t) = \dfrac{8000}{1 + 7e^{-\frac{t}{6}}}$. **3**

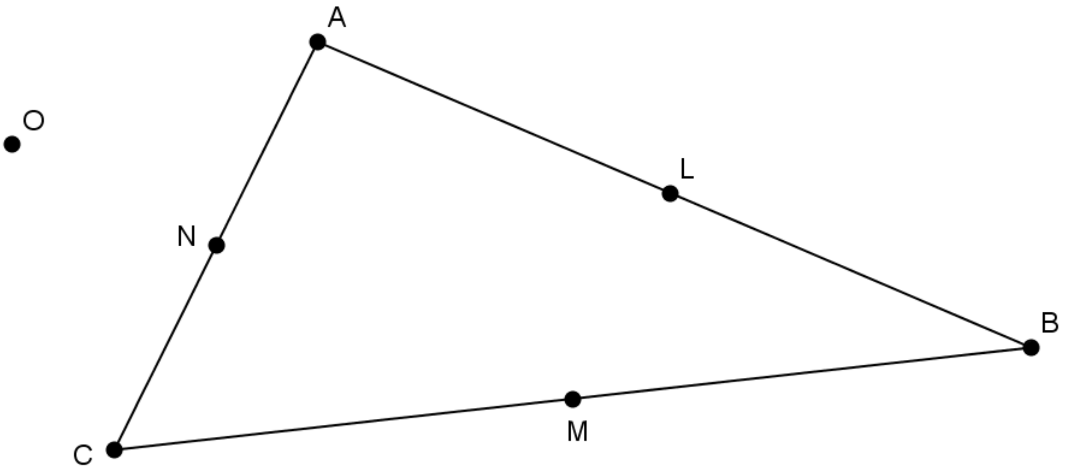
(You may use the fact that $\dfrac{Q}{P(Q-P)} = \dfrac{1}{P} + \dfrac{1}{Q-P}$)

(ii) If instead the initial population P_0 is 12 000 bacteria, describe what would have happened to the population as t increases. **1**

Question 14 continues on the next page

Question 14 continued

(c) The diagram shows O as the origin and a triangle with vertices *A*, *B* and *C*. Let the vectors $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. *L*, *M* and *N* are the midpoints of $\overrightarrow{AB}$, $\overrightarrow{CB}$ and $\overrightarrow{AC}$ respectively.



(i) Show that quadrilateral *LMNA* is a parallelogram. **3**

(ii) Deduce that a line through the midpoints of two sides of triangle is half the length of the third side. **1**

(d) The polynomial $g(x) = 4x^3 - 3x + 1$ passes through the point $(1,\ 2)$.

(i) Show that $g(x)$ has a root of multiplicity of 2. **2**

(ii) Find the gradient of the tangent to $f(x) = \dfrac{g^{-1}(x)}{x}$ at the point where $x = 2$. **3**

End of paper

Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial 2024 Solutions

Multiple Choice

Solutions	Marker's Comments
Question 1 $\underset{\sim}{a} = 3\underset{\sim}{i} + \underset{\sim}{j}$ $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j}$ $\lvert\underset{\sim}{a}\rvert = \sqrt{9+1}$, $\lvert\underset{\sim}{b}\rvert = \sqrt{4+1}$ $= \sqrt{10}$ $= \sqrt{5}$ $\cos\theta = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert} = \dfrac{6-1}{\sqrt{10}\sqrt{5}} = \dfrac{5}{\sqrt{50}} = \dfrac{\sqrt{25}}{\sqrt{50}} = \dfrac{1}{\sqrt{2}}$ $\cos\theta = \dfrac{1}{\sqrt{2}}$ $\theta = \dfrac{\pi}{4} \quad \therefore B$	
Question 2 Since $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$ $2\cos^2\theta = 1 + \cos 2\theta$ $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ $\therefore \cos^2 4x = \frac{1}{2}(1+\cos 8x)$ then $\int \cos^2 4x\, dx$ $= \int \dfrac{1+\cos 8x}{2}\, dx \quad \therefore B$	
Question 3 $P(x) = x^3 - 3x^2 + 2x + 3$ $P(-1) = (-1)^3 - 3(-1)^2 + 2(-1) + 3$ $= -1 - 3 - 2 + 3$ $= -3 \quad \therefore A$	

Solutions	Marker's Comments
Question 4 $\frac{dy}{dx}=\frac{1}{xy}$ $y.\,dy=\frac{1}{x}dx$ $\int y.\,dy=\int\frac{1}{x}dx$ $\frac{y^2}{2}=\ln\lvert x\rvert+c_1$ $y^2=2\ln\lvert x\rvert+c_2 \quad \therefore D$	
Question 5 $\overrightarrow{OA}\bullet\overrightarrow{OB}=0$ $\therefore\overrightarrow{OA}\perp\overrightarrow{OB} \quad \therefore C$	
Question 6 $\left(3x-\frac{2}{x^2}\right)^9$ has general term ${}^9C_r\;\;3x^{\;9-r}\left(\frac{-2}{x^2}\right)^r$ $={}^9C_r.3^{9-r}.x^{9-r}.\;-1^{\;r}.2^r.x^{-2r}$ $={}^9C_r.3^{9-r}.2^r.\;-1^{\;r}.x^{9-r}.x^{-2r}$ $={}^9C_r.3^{9-r}.2^r.\;-1^{\;r}.x^{9-3r}$ Constant term when $9-3r=0$ $3r=9$ $r=3$ ${}^9C_3.3^6.2^3.\;-1^{\;3}.x^0$ $=-{}^9C_3.3^6.2^3\;but\;{}^9C_3={}^9C_6$ $=-{}^9C_6.3^6.2^3 \quad \therefore D$	

Solutions	Marker's Comments
Question 7 $\sin x+\sqrt{3}\cos x=A\cos(x+\theta)$ $\sin x+\sqrt{3}\cos x=A\cos x\cos\theta-A\sin x\sin\theta$ so $A\sin\theta=-1$ $A\cos\theta=\sqrt{3}$ $\therefore\tan\theta=\frac{-1}{\sqrt{3}}$ $\theta=\frac{-\pi}{6}$ and $A^2\sin^2\theta+A^2\cos^2\theta=(-1)^2+(\sqrt{3})^2$ $A^2(\sin^2\theta+\cos^2\theta)=1+3$ $A^2=4$ $A=2$ $\therefore\sin x+\sqrt{3}\cos x=2\cos(x+\frac{-\pi}{6})$ $\sin x+\sqrt{3}\cos x=2\cos\left(x-\frac{\pi}{6}\right)\quad\therefore A$	
Question 8 $1\times7\times8!=282\,240$ Or $9!-2\times8!=282\,240\quad\therefore C$	

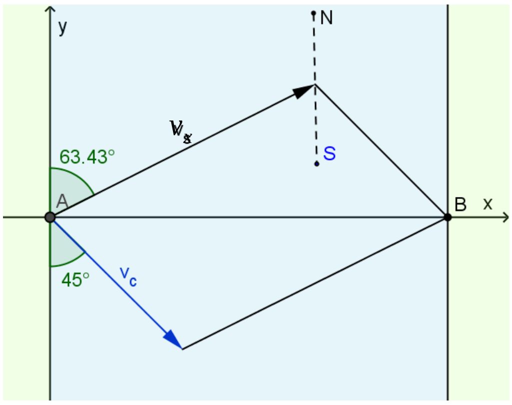
Solutions	Marker's Comments
Question 9 $y=\sqrt{x+1}$ is equivalent to $y^2=x+1$ with $D: x\ge -1$, $R: y\ge 0$ (A) $x=t^2-1, y=t$ *for* $t\ge 0$ $x=y^2-1 \quad D: x\ge -1$ $y^2=x+1 \quad R: y\ge 0$ which is equivalent to $y=\sqrt{x+1}$. (B) $x=t, y=\sqrt{t+1}$ *for* $t\ge 1$ which is equivalent to $y=\sqrt{x+1}$ with $D: x\ge +1$, $R: y\ge 0$ and so is NOT equivalent $y=\sqrt{x+1}$. $\therefore B$. (C) $x=t-1, y=\sqrt{t}$ *for* $t\ge 0$ $t=x+1$ $\therefore y=\sqrt{x+1}$ with $D: x\ge -1$, $R: y\ge 0$ which is equivalent to $y=\sqrt{x+1}$. (D) $x=t-2, y=\sqrt{t-1}$ *for* $t\ge 1$ $t=x+2$ $y=\sqrt{(x+2)-1}$ $y=\sqrt{x+1}$ with $D: x\ge -1$, $R: y\ge 0$ which is equivalent to $y=\sqrt{x+1}$.	

Question 10	
Domain of $y=\sin^{-1}x$ is $-1\le x\le 1$ so domain of $y=\sin(\sin^{-1}x)$ is also $-1\le x\le 1$. **A**	

SECTION II

Solutions	**Marker's Comments**
Question 11 (a) $\left(2\underset{\sim}{i}+3\underset{\sim}{j}\right)-\left(\underset{\sim}{i}-2\underset{\sim}{j}\right)=\underset{\sim}{i}+5\underset{\sim}{j}$	Very well done
(b) Show that $\sin x-\tan\frac{x}{2}=\tan\frac{x}{2}\cos x$ $LHS=\sin x-\tan\frac{x}{2}$ $=\frac{2t}{1+t^2}-t$ $=\frac{2t-t(1+t^2)}{1+t^2}$ $=\frac{2t-t-t^3}{1+t^2}$ $=\frac{t-t^3}{1+t^2}$ $=\frac{t(1-t^2)}{1+t^2}$ $=t\times\frac{1-t^2}{1+t^2}$ $=\tan\frac{x}{2}\times\cos x$ $LHS=RHS$	Done well by most students. Students need to take care to set their working out appropriately and show all steps for a 'show that' question.
Question 11 (c) $\int x\sqrt{x+2}\,dx$ using $u=x+2$ $\therefore x=u-2$	Done well by most students.

$\frac{du}{dx}=1$ $du=dx$ $\int x\sqrt{x+2}\ dx$ $=\int (u-2).u^{\frac{1}{2}}.\ du$ $=\int (u^{\frac{3}{2}}-2u^{\frac{1}{2}}).\ du$ $=\frac{u^{\frac{5}{2}}}{\frac{5}{2}}-\frac{2u^{\frac{3}{2}}}{\frac{3}{2}}+c$ $=\frac{2\sqrt{u^5}}{5}-\frac{2\times 2\sqrt{u^3}}{3}+c$ $=\frac{2\sqrt{(x+2)^5}}{5}-\frac{4\sqrt{(x+2)^3}}{3}+c$ *or* $=\frac{2\left(\sqrt{x+2}\right)^5}{5}-\frac{4\left(\sqrt{x+2}\right)^3}{3}+c$	There were a considerable number if students who left their answer in terms of *u*. It is essential to rewrite your answer in terms of *x*.
(d) $8-\lvert 2x-1\rvert\leq 5$ $-\lvert 2x-1\rvert\leq -3$ $\lvert 2x-1\rvert\geq 3$ $2x-1\leq -3 \quad or \quad 2x-1\geq 3$ $2x\leq -2 \qquad 2x\geq 4$ $x\leq -1 \qquad x\geq 2$ or $2x-1\ ^{2}\geq 3^{2}$ $4x^2-4x+1\geq 9$ $4x^2-4x-8\geq 0$ $4(x^2-x-2)\geq 0$ $4(x-2)(x+1)\geq 0$ $x\leq -1,\ x\geq 2$	Generally done well. Most students used the first method and were generally successful. Not many students used the second method involving squaring both sides. Many of those that did use this method went on to make errors.
Question 11 (e) $\frac{dV}{dt}=3$ and $V=\pi r^2 h$ $V=\pi\times 2^2 h$ $V=4\pi h$ $\frac{dV}{dh}=4\pi$	There were errors with the formula for volume $V\neq\frac{1}{3}\pi r^2 h$ $V\neq 4\pi r^2 h$

$\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}$ $=\frac{1}{4\pi}\times 3$ $\frac{dh}{dt}=\frac{3}{4\pi}\,m/\min$	Many students made an error at the end showing a basic lack of understanding $\frac{dh}{dt}=\frac{3}{4\pi}\;m/\min$ $\frac{dh}{dt}=\frac{3}{4}\times\frac{1}{\pi}m/\min$ $\frac{dh}{dt}\neq\frac{3}{4}\pi\;m/\min$ ISE =Ignore Subsequent Error
(f) $\int_0^1\frac{dx}{x^2+3}$ $=\frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{x}{\sqrt{3}}\right]_0^1$ $=\frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{1}{\sqrt{3}}-\tan^{-1}\frac{0}{\sqrt{3}}\right]$ $=\frac{1}{\sqrt{3}}\left[\frac{\pi}{6}-0\right]$ $=\frac{\pi}{6\sqrt{3}}\quad or\quad\frac{\pi\sqrt{3}}{18}$	Done well. Only two students confused it with log
Question 11 (g) Speed of current: $\lvert v_c\rvert=1.2$ ms^{-1} Bearing of current: 135° T  $\lvert v_x\rvert=3$ m/s $\lvert v_{AB}\rvert_x^2=(3)^2+(1.2)^2-2(3)(1.2)\cos(63.43+45)^\circ$ $=9+1.44-7.2\cos108.43^\circ$ $=12.7162...$ $\therefore\lvert v_{AB}\rvert_x=3.5659...$ m/s Swim speed of y = 2.4 m/s Ratio of swim speed of y is to $x=\frac{2.4}{3}$	1 mark awarded here. Some students achieved this mark

$=\frac{4}{5}$ $\therefore \lvert v_{AB}\rvert_y = \frac{4}{5}\times 3.5659...$ $= 2.85278\ldots$ $= 2.85$ m/s $(2.85)^2 = (2.4)^2 + (1.2)^2 - 2(2.4)(1.2)\cos(\theta+45)^\circ$ $\cos(\theta+45)^\circ = \frac{(2.4)^2+(1.2)^2-(2.85)^2}{2(2.4)(1.2)}$ $\cos(\theta+45)^\circ = -0.16015...$ $\theta + 45^\circ = 99.37623...^\circ$ $\therefore \theta = 54.37623...^\circ$ $\therefore \theta = 54.38^\circ$ (to 2 d.p.) The bearing of swimmer Y from A is $054.38^\circ T = 054^\circ 23' T$	No students dealt with the relationship between the speeds of the two swimmers
Alternative solution Ocean current is given by vector $\underset{\sim}{c} = 1.2\cos 45^o \underset{\sim}{i} - 1.2\sin 45^o \underset{\sim}{j}$ $\underset{\sim}{c} = 1.2\frac{1}{\sqrt{2}}\underset{\sim}{i} - 1.2\frac{1}{\sqrt{2}}\underset{\sim}{j}$ $\underset{\sim}{c} = \sqrt{0.72}\underset{\sim}{i} - \sqrt{0.72}\underset{\sim}{j}$ $\underset{\sim}{c} \cong 0.8485\underset{\sim}{i} - 0.8485\underset{\sim}{j}$ Swimmer X is given by vector $\underset{\sim}{x} = 3\cos 26.57^o \underset{\sim}{i} + 3\sin 26.57^o \underset{\sim}{j}$	OR
Swimmer Y is given by vector $\underset{\sim}{y} = 2.4\cos\theta^o \underset{\sim}{i} + 2.4\sin\theta^o \underset{\sim}{j}$	1 mark awarded here. Some students achieved this mark
Swimmer X swims from A to B with velocity $\lvert v_{AB}\rvert_x = \sqrt{\left(3\cos 26.57 + \sqrt{0.72}\right)^2 + \left(3\sin 26.57 - \sqrt{0.72}\right)^2}$ $\lvert v_{AB}\rvert_x = \sqrt{12.71624993...}$ $\lvert v_{AB}\rvert_x = 3.565985128...\, m/s$	No students dealt with the relationship between the speeds of the two swimmers
Swimmer Y swims at $\frac{2.4}{3} = \frac{4}{5}$ the rate of swimmer X $\frac{4}{5}\times\lvert v_{AB}\rvert_x = \lvert v_{AB}\rvert_y$ $\frac{4}{5}\times 3.5659... = \sqrt{\left(2.4\cos\theta + \sqrt{0.72}\right)^2 + \left(2.4\sin\theta - \sqrt{0.72}\right)^2}$	Some students were awarded a second mark here

$\left(\frac{4}{5}\times 3.5659...\right)^2 = 2.4^2\cos^2\theta + 2\times 2.4\cos\theta\sqrt{0.72} + 0.72 + 2.4^2\sin^2\theta - 2\times 2.4\sin\theta\sqrt{0.72} + 0.72$ $\frac{16}{25}\times 12.71624993... = 2.4^2(\cos^2\theta + \sin^2\theta) + 2\times 0.72 + 2\times 2.4\sqrt{0.72}(\cos\theta - \sin\theta)$ $\frac{16}{25}\times 12.71624993... = 2.4^2\times 1 + 1.44 + 4.8\sqrt{0.72}(\cos\theta - \sin\theta)$ $(\cos\theta - \sin\theta) = \dfrac{\frac{16}{25}\times 12.71624993... - 5.76 - 1.44}{4.8\sqrt{0.72}}$ $(\cos\theta - \sin\theta) = 0.2303989489...$ $(\cos\theta - \sin\theta)^2 = 0.2303989489...^2$ $\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta = 0.05308367568...$ $\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta = 0.05308367568...$ $1 - \sin 2\theta = 0.05308367568...$ $\sin 2\theta = 0.9469...$ $2\theta = 71.2475...$ $\theta = 35.6237...$ Bearing is $90^o - 35.62...^o$ $= 54.376..^o$ $= 54^o 23'$ Common error was to work with the ocean current and swimmer Y to reach due east which would mean the vertical component needs to equal zero $2.4\sin\theta - 1.2\sin 45 = 0$ $2.4\sin\theta - \sqrt{0.72} = 0$ $2.4\sin\theta = \sqrt{0.72}$ $\sin\theta = \dfrac{\sqrt{0.72}}{2.4}$ $\sin\theta = 0.3535533906...$ $\theta = 20.70481105^o$ Bearing is $90^o - 20.70481...^o$ $= 69.295..^o$	Some students were awarded 1 mark for dealing with some of the concepts to get this incorrect bearing
Question 12 (a) $f(x) = 2^x - 3$ $x = 0, y = 1 - 3 = -2$ $y = 0, 2^x = 3, x = \dfrac{\ln 3}{\ln 2}$ $\therefore$ the y-intercept is $(0, -2)$, the x-intercept is $\left(\dfrac{\ln 3}{\ln 2}, 0\right)$	

<table>
<tr>
<td>

$f(x) = 2^x - 3$

$D: x \in \mathbb{R}, \quad R: y > -3$

$x = 2^y - 3$

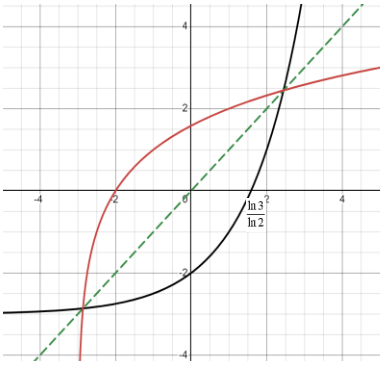
$2^y = x + 3$

$\ln 2^y = \ln|x+3|$

$y = \dfrac{\ln|x+3|}{\ln 2}$

$\therefore f^{-1}(x) = \dfrac{\ln(x+3)}{\ln 2}$

$D: x > -3, \quad y \in \mathbb{R}$



</td>
<td>

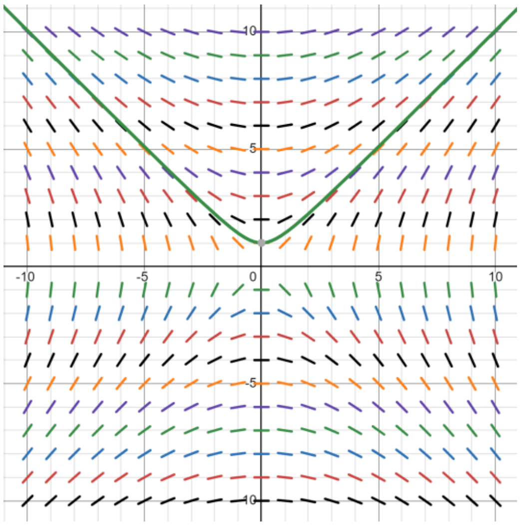
Many students didn't read the question carefully and didn't give the answer in natural log.

Some students didn't realise that $x \neq -3$.

Some mistakes with logs:
$\dfrac{\ln|x+3|}{\ln 2} \neq \ln\left(\dfrac{|x+3|}{2}\right)$.

</td>
</tr>
<tr>
<td>

(b)



</td>
<td>

Some students didn't draw the curve passing (0,1).

Some students draw more solutions.

</td>
</tr>
<tr>
<td>

Solutions

</td>
<td>

Marker's Comments

</td>
</tr>
<tr>
<td>

Question 12

(c) $\dfrac{dT}{dt} = k(T-18)$

$\displaystyle\int \frac{dT}{(T-18)} = \int k\,dt$

$\ln|T-18| = kt + C$

$T - 18 = e^{kt+C}$

$T - 18 = Ae^{kt} \quad (A = e^C)$

$T = 18 + Ae^{kt}$

</td>
<td>

Students need to show sufficient workings to justify how they find the value of k.

</td>
</tr>
</table>

$t=0, T=98$ $98=18+Ae^{k(0)}$ $A=80$ $\therefore T=18+80e^{kt}$ $t=5, T=58$ $58=18+80e^{k(5)}$ $80e^{5k}=40$ $e^{5k}=\frac{1}{2}$ $5k=\ln\frac{1}{2}$ $k=\frac{\ln\frac{1}{2}}{5}$ $\therefore k=\frac{-\ln 2}{5}$ (ii) $T=18+80e^{\frac{-\ln 2}{5}t}$ $T=20,$ $20=18+80e^{\frac{-\ln 2}{5}t}$ $2=80e^{\frac{-\ln 2}{5}t}$ $\frac{1}{40}=e^{\frac{-\ln 2}{5}t}$ $\ln\frac{1}{40}=\frac{-\ln 2}{5}t$ $-\ln 40=\frac{-\ln 2}{5}t$ $t=\frac{5\ln 40}{\ln 2}$ $=26.60964....$ ≈ 27 min $\therefore$ It takes approximately 27 minutes for the egg to drop to 20°C.	Generally well done.
(d)	Students need to show sufficient workings, e.g $\sin\theta+\sin 5\theta=2\sin\left(\frac{\theta+5\theta}{2}\right)\cos\left(\frac{\theta-5\theta}{2}\right)$ Or $\sin\theta+\sin 5\theta=\sin(3\theta-2\theta)+\sin(3\theta+2\theta)$

Solution	Marks	Marker's Comments
$LHS = \frac{\sin\theta+\sin3\theta+\sin5\theta}{\cos\theta+\cos3\theta+\cos5\theta}$ $LHS = \frac{(\sin\theta+\sin5\theta)+\sin3\theta}{(\cos\theta+\cos5\theta)+\cos3\theta}$ $LHS = \frac{2\sin\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right)+\sin3\theta}{2\cos\left(\frac{5\theta+\theta}{2}\right)\cos\left(\frac{5\theta-\theta}{2}\right)+\cos3\theta}$ $LHS = \frac{2\sin3\theta\cos2\theta+\sin3\theta}{2\cos3\theta\cos2\theta+\cos3\theta}$ $LHS = \frac{\sin3\theta\,(2\cos2\theta+1)}{\cos3\theta\,(2\cos2\theta+1)}$ $LHS = \frac{\sin3\theta}{\cos3\theta}$ $LHS = \tan3\theta$ $LHS = RHS$	2 Marks Correct proof 1 Mark Uses sums to products formula to demonstrate that $LHS = \frac{2\sin3\theta\cos2\theta+\sin3\theta}{2\cos3\theta\cos2\theta+\cos3\theta}$	Generally well done.
$\frac{\sin\theta+\sin3\theta+\sin5\theta}{\cos\theta+\cos3\theta+\cos5\theta} = 1, 0 \le \theta \le 2\pi$ $\tan3\theta = 1$ $3\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$ $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$	1 Mark All answers correct	

Solutions	Marker's Comments
Question 13 (a) $V = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2 y - y^2 \; dy$ $V = \pi\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec^2 y - 1 - y^2 \; dy$ $V = \pi\left[\tan y - y - \frac{y^3}{3}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ $V = \pi\left[\tan\frac{\pi}{3} - \frac{\pi}{3} - \frac{\frac{\pi}{3}^3}{3} - \left(\tan\frac{\pi}{4} - \frac{\pi}{4} - \frac{\frac{\pi}{4}^3}{3}\right)\right]$ $V = \pi\left[\sqrt{3} - \frac{\pi}{3} - \frac{\pi^3}{81} - \left(1 - \frac{\pi}{4} - \frac{\pi^3}{192}\right)\right]$ $V = \pi\left[\sqrt{3} - 1 - \frac{\pi}{12} - \frac{\pi^3}{81} - \left(\frac{\pi^3}{192}\right)\right]$ $V \approx 0\cdot78\, units^3$	Mostly well done question 1 mark for correct integral of volume set up 2nd mark for correct sub of tan squared. 3rd mark for correct integration and/or substitution. 4th mark for correct numerical expression or 0.78 units cubed Common errors where -forgetting the minus y^2 -not knowing how to deal with the $\tan^2 y$, quite a few converted to $\frac{\sin^2 y}{\cos^2 y}$ which usually led to errors.

<table>
<tr>
<td></td>
<td>It was good to se no short cuts with the substitution were taken because this was a correct numerical expression (CNE) which attained full marks if done correctly even if an error occurred in the final answer.</td>
</tr>
<tr>
<td>

(b)

for $n=1$, LHS$=\frac{2}{1\times 3}=\frac{2}{3}$

$$\text{RHS}=\frac{3}{2}-\frac{2(1)+3}{(1+1)(1+2)}$$

$$=\frac{3}{2}-\frac{5}{6}$$

$$=\frac{9}{6}-\frac{5}{6}$$

$$=\frac{4}{6}$$

$$=\frac{2}{3}=RHS$$

$\therefore$ true for $n=1$

Assume true for $n=k$,

$$\frac{2}{1\times 3}+\frac{2}{2\times 4}+\frac{2}{3\times 5}+...+\frac{2}{k(k+2)}=\frac{3}{2}-\frac{2k+3}{(k+1)(k+2)}$$

Required to prove true for $n=k+1$,

$$i.e.\ \frac{2}{1\times 3}+\frac{2}{2\times 4}+\frac{2}{3\times 5}+...+\frac{2}{k(k+2)}+\frac{2}{(k+1)(k+1+2)}=\frac{3}{2}-\frac{2(k+1)+3}{(k+1+1)(k+1+2)}=\frac{3}{2}-\frac{2k+5}{(k+2)(k+3)}$$

Proof:

$$\text{LHS}=\frac{2}{1\times 3}+\frac{2}{2\times 4}+\frac{2}{3\times 5}+...+\frac{2}{k(k+2)}+\frac{2}{(k+1)(k+3)}$$

$$=\frac{3}{2}-\frac{2k+3}{(k+1)(k+2)}+\frac{2}{(k+1)(k+1+2)}\qquad(\text{by assumption})$$

$$=\frac{3}{2}-\left(\frac{(2k+3)(k+3)-2(k+2)}{(k+1)(k+2)(k+3)}\right)$$

$$=\frac{3}{2}-\frac{2k^2+6k+3k+9-2k-4}{(k+1)(k+2)(k+3)}$$

$$=\frac{3}{2}-\frac{2k^2+7k+5}{(k+1)(k+2)(k+3)}$$

$$=\frac{3}{2}-\frac{(2k+5)(k+1)}{(k+1)(k+2)(k+3)}$$

$$=\frac{3}{2}-\frac{(2k+5)(k+1)}{(k+1)(k+2)(k+3)}$$

$$=\frac{3}{2}-\frac{2k+5}{(k+2)(k+3)}=RHS$$

If true for n=k, proven true for n=k+1.

</td>
<td>

Please be careful with induction and do not gloss over step 1. Substitution needs to be shown in RHS to PROVE it is true for n=1

1 mark awarded here.

This statement needs to be made for n=k

A correct statement needs to be made for n=k+1.
This was awarded the second mark.

This line here is where many made an error. The + becoming a – when writing the fractions with a common denominator. You can tell when students work back whether they realised they made a mistake or simply tried to fudge the answer at the end.

</td>
</tr>
</table>

Since true for n=1, true for n=1+1=2, n=2+1=3, …
Therefore, true for any positive integer n.

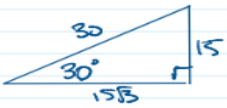
Usually, no mark is given for the conclusion, but this may vary year to year. It must be there to finish off the inductive process. It may lead to loss of mark if not written or not properly written.

c)

c. i $\underset{\sim}{r}(t) = 15\sqrt{3}t\,\underset{\sim}{i} + (15t - 4.9t^2)\,\underset{\sim}{j}$

$\dot{\underset{\sim}{r}}(t) = 15\sqrt{3}\,\underset{\sim}{i} + (15 - 9.8t)\,\underset{\sim}{j}$

$\dot{\underset{\sim}{r}}(0) = 15\sqrt{3}\,\underset{\sim}{i} + 15\,\underset{\sim}{j}$



initial velocity is $30\,ms^{-1}$, with an angle of 30°.

1 – correct expr. for velocity or equiv.

2 – correct ans

Generally, well done. Some did not find the initial velocity value of 30m/s but found the initial velocity vector. This was allowed as it may have not been quite clear as to what initial velocity was asked for.

ii. max height when $\dot{y} = 0$

vertical velocity $\dot{y} = 15 - 9.8t$

$\Rightarrow t = 1.5306$

$y = 15t - 4.9t^2$

$= 15(1.5306) - 4.9(1.5306)^2$

$= 11.47959$

$= 11.48\ m$

1 – time of max height

2 – correct ans

Generally, well done

iii time of flight when $y = 0$

$15t - 4.9t^2 = 0$

$t(15 - 4.9t) = 0$

$\therefore t = 3.061\ sec$

1 – correct ans

Generally, well done

[redacted]

iv. at $t = 3.061$,

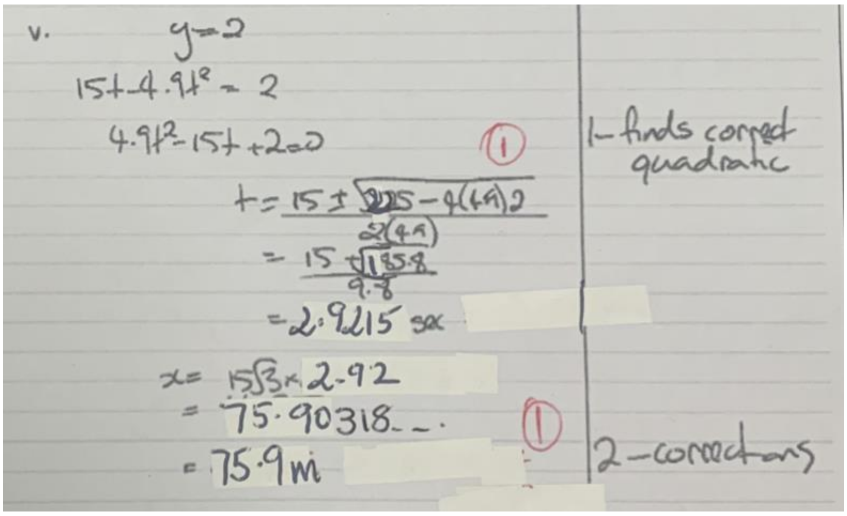
$x = 15\sqrt{3}(3.061)$

$= 79.52711$

$= 79.5\ m$

1 – correct ans

Generally, well done



Generally, well done

Solutions	**Marker's Comments**
Question 14 (a)	Mostly did well.
Question 14 (b)	22% got full marks. Many students (14%) should copy the given expression before they edit or modify as their first line of working. 14% did not "show" appropriate working.

Question 14

(a) The roots of $x^3+3x^2-4=0$ are α , β and γ .

$$\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$$

$$=\frac{-b}{a}\div\frac{-d}{a}$$

$$=\frac{-3}{1}\div\frac{-(-4)}{1} \quad ①$$

$$=-3\times\frac{1}{4}$$

$$=\frac{-3}{4} \quad ①$$

Mostly did well.

(b)

(i) $$\frac{dP}{dt}=\frac{P}{6}\left(1-\frac{P}{8000}\right)$$

$$\frac{dP}{dt}=\frac{P}{6}\left(\frac{8000-P}{8000}\right)$$

$$\left(\frac{8000}{P(8000-P)}\right)dP=\frac{1}{6}\,dt$$

$$\int\left(\frac{8000}{P(8000-P)}\right)dP=\frac{1}{6}\int dt$$

Using $$\frac{Q}{P(Q-P)}=\frac{1}{P}+\frac{1}{Q-P}$$

$$\int\frac{1}{P}+\frac{1}{8000-P}\,dP=\frac{1}{6}\int dt$$

$$\int\frac{1}{P}-\frac{-1}{8000-P}\,dP=\frac{1}{6}\int dt$$

$$\ln|P|-\ln|8000-P|=\frac{1}{6}t+C \quad ①$$

22% got full marks.

Many students (14%) should copy the given expression before they edit or modify as their first line of working.

14% did not "show" appropriate working.

$$\ln\left|\frac{P}{8000-P}\right| = \frac{1}{6}t + C$$

$$\therefore \left|\frac{P}{8000-P}\right| = e^{\frac{t}{6}+C}$$

$$\therefore \left|\frac{P}{8000-P}\right| = e^{\frac{t}{6}}e^{C}$$

$$\therefore \frac{P}{8000-P} = Ae^{\frac{t}{6}} \quad \text{where } A = \pm e^{C}$$

When $t = 0$, $P = 1000$, $\dfrac{(1000)}{8000-(1000)} = Ae^{\frac{(0)}{6}}$

$$\frac{1000}{7000} = Ae^{0}$$

$$\therefore A = \frac{1}{7}$$ ①

$$\therefore \frac{P}{8000-P} = \frac{1}{7}e^{\frac{t}{6}}$$

$$P = \frac{1}{7}e^{\frac{t}{6}}(8000-P)$$

$$7P = e^{\frac{t}{6}}(8000-P)$$

$$7P = 8000e^{\frac{t}{6}} - Pe^{\frac{t}{6}}$$

$$7P + Pe^{\frac{t}{6}} = 8000e^{\frac{t}{6}}$$

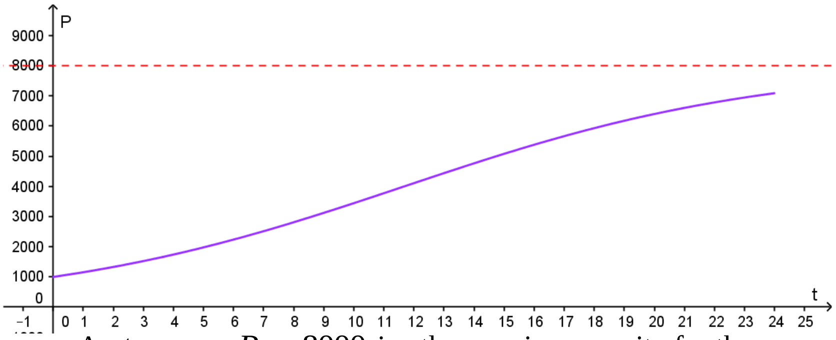
$$P\left(7 + e^{\frac{t}{6}}\right) = 8000e^{\frac{t}{6}}$$

$$P = \frac{8000e^{\frac{t}{6}}}{7 + e^{\frac{t}{6}}}$$

$$P = \frac{\left(\dfrac{8000e^{\frac{t}{6}}}{e^{\frac{t}{6}}}\right)}{\left(\dfrac{7 + e^{\frac{t}{6}}}{e^{\frac{t}{6}}}\right)}$$ ①

$$\therefore P = \frac{8000}{7e^{\frac{-t}{6}} + 1}$$ (as required)

(b) (ii) Method 1: Graphing $P(t)$.



As $t \to \infty$, $P \to 8000$ i.e. the carrying capacity for the population is 8000. Hence when initial population is 12000, the population will decay until it reaches the asymptotic population of 8000 as time increases.

Method 2: Using limits of $P(t)$.

As $t \to \infty$, $e^{-\frac{t}{6}} \to 0$

$$\lim_{t\to\infty} P = \lim_{t\to\infty} \frac{8000}{7e^{-\frac{t}{6}}+1}$$

$$= \frac{8000}{7(0)+1}$$

$$= \frac{8000}{1}$$ i.e. carrying capacity of the population.

Hence when initial population is 12000, the population will decay until it reaches the asymptotic population of 8000 as time increases.

Method 3: Using $\frac{dP}{dt} = \frac{P}{6}\left(1-\frac{P}{8000}\right)$

So $\frac{dP}{dt} = \frac{P}{6}\left(1-\frac{P}{8000}\right)$ and when $P_0 = 12000$

$$\frac{dP_0}{dt} = \frac{12000}{6}\left(1-\frac{12000}{8000}\right)$$

$$\frac{dP_0}{dt} = 2000\left(1-\frac{3}{2}\right)$$

$$\frac{dP_0}{dt} = 2000\left(-\frac{1}{2}\right)$$

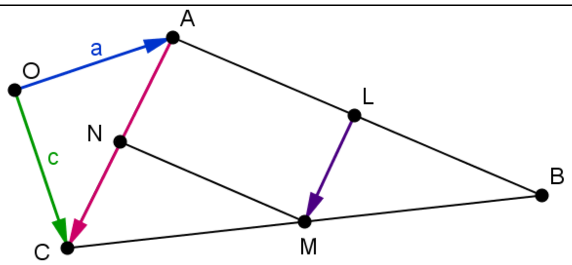
$\frac{dP_0}{dt} = -6000$ bacteria/hr < 0 i.e. the population will decrease until $\frac{dP}{dt} \to 0$ as $t \to \infty$, $P \to 8000$ bacteria which is the carrying capacity of the population.

26% got full marks.
(Accept simple statement such as:

The population will decrease to 8000).

Many students are not understanding what the horizontal asymptote implies from the equation in practical sense, hence are missing the limit to population is 8000 due to the carrying capacity of the petri dish.

(c)



46% got full marks.

A few students need to pay attention to the direction of vectors.

Note: Students only need to show that one pair of opposite vectors

(i) $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}$ $\therefore \overrightarrow{AC}=\underset{\sim}{c}-\underset{\sim}{a}$ $\overrightarrow{AN}=\frac{1}{2}\overrightarrow{AC}$ $\therefore \overrightarrow{AN}=\frac{1}{2}(\underset{\sim}{c}-\underset{\sim}{a})$ ① $\overrightarrow{LM}=\overrightarrow{LB}+\overrightarrow{BM}$ $=\frac{1}{2}\overrightarrow{AB}-\frac{1}{2}\overrightarrow{CB}$ $=\frac{1}{2}(\overrightarrow{OB}-\overrightarrow{OA})-\frac{1}{2}(\overrightarrow{OB}-\overrightarrow{OC})$ $=\frac{1}{2}\overrightarrow{OB}-\frac{1}{2}\underset{\sim}{a}-\frac{1}{2}\overrightarrow{OB}+\frac{1}{2}\underset{\sim}{c}$ $\overrightarrow{LM}=-\frac{1}{2}\underset{\sim}{a}+\frac{1}{2}\underset{\sim}{c}$ $=\frac{1}{2}(\underset{\sim}{c}-\underset{\sim}{a})$ ① $\therefore \overrightarrow{LM}=\overrightarrow{AN}$ (a pair of equal paprallel opposite side) ① Hence *ALMN* is a parallelogram.	are equal to prove for a quadrilateral is a parallelogram.

Solutions	Marker's Comments
c)(ii) $\overrightarrow{AN}=\frac{1}{2}\overrightarrow{AC}$ $\lvert\overrightarrow{AN}\rvert=\frac{1}{2}\lvert\overrightarrow{AC}\rvert$ and $\overrightarrow{LM}=\overrightarrow{AN}$ i.e. $\lvert\overrightarrow{LM}\rvert=\lvert\overrightarrow{AN}\rvert$ $\therefore \lvert\overrightarrow{LM}\rvert=\frac{1}{2}\lvert\overrightarrow{AC}\rvert$ i.e. the length of the line through the midpoints of two sides of triangle is half the length of the third side. ① **Question 14** d)(i) $g(x)=4x^3-3x+1$ $g'(x)=12x^2-3$ For multiplicity of 2, $g'(x)=0$ $12x^2-3=0$ $3(4x^2-1)=0$ $4x^2-1=0$ $(2x-1)(2x+1)=0$	Many students did not use part (i) as part of the answer for this question. Quite poorly done.

$\therefore x = \pm\frac{1}{2}$ ① $g\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right) + 1 \neq 0$ $g\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right) + 1 = 0$ ① $\therefore (2x-1)^2$ is a factor. $g(-1) = 4(-1)^3 - 3(-1) + 1 = 0$ $\therefore g(x) = (2x-1)^2(x+1)$ $\therefore g(x)$ has a multiplicity of 2. Or $\left(x - \frac{1}{2}\right)^2$ and then use division transformation $x^2 - x + \frac{1}{4}\ \overline{)\ 4x^3 \quad - 3x + 1}$ quotient $4x + 4$ $\underline{4x^3 - 4x^2 + x}$ $4x^2 - 4x + 1$ $\underline{4x^2 - 4x + 1}$ 0 $g(x) = 4(x+1)(x-\frac{1}{2})^2$ has a multiplicity of 2	There are two solutions for $g'(x) = 12x^2 - 3$, not just one solution of $x = \frac{1}{2}$. So students that assumed that $x = \frac{1}{2}$ is the only solution and tested $g\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right) + 1 = 0$, can only be assigned 1 mark. Students that answered with $g\left(\frac{1}{2}\right) = 0$ without showing substitution is not awarded a mark.
(ii) $g(x) = 4x^3 - 3x + 1$ at (1, 2) i.e. $g(1) = 2$ $g^{-1}(2) = 1$ $f(x) = \frac{g^{-1}(x)}{x}$ ① $f'(x) = \frac{(g^{-1})'(x)x - g^{-1}(x)}{x^2}$ and if $g^{-1}(x) = y$ $x = g(y)$ $\frac{dx}{dy} = g'(y)$ $\frac{dy}{dx} = \frac{1}{g'(y)}$ $\frac{d}{dx}g^{-1}(x) = \frac{1}{g'\left(g^{-1}(x)\right)}$ $(g^{-1})'(x) = \frac{1}{g'\left(g^{-1}(x)\right)}$	Poorly done. Many students did not know what to do.

$$f'(x)=\frac{\frac{1}{g'\left(g^{-1}(x)\right)}x-g^{-1}(x)}{x^2}$$

$$f'(2)=\frac{\frac{1}{g'\left(g^{-1}(2)\right)}(2)-g^{-1}(2)}{(2)^2}$$

$$f'(2)=\frac{\frac{2}{g'((1))}-1}{4}$$ ①

$$f'(2)=\frac{\left(\frac{2}{12(1)^2-3}-1\right)}{4}$$

$$f'(2)=\frac{\frac{2}{9}-1}{4}$$

$$f'(2)=\frac{\left(-\frac{7}{9}\right)}{4}$$

$$\therefore f'(2)=\frac{-7}{36}$$ ①

Hence the gradient of the tangent is $-\frac{7}{36}$

Alternative notation:

$$f'(x)=\frac{x\times\frac{d}{dx}\ g^{-1}(x)\ -g^{-1}(x)\times 1}{x^2}$$ ①

$$f'(2)=\frac{2\times\frac{d}{dx}\ g^{-1}(2)\ -g^{-1}(2)\times 1}{2^2}$$

$g:\quad y=4x^3-3x+1$ *passes through* $1,2$

$g^{-1}:\quad x=4y^3-3y+1$ *passes through* $2,1$

$$\frac{dx}{dy}=12y^2-3$$

$$\therefore\frac{dy}{dx}=\frac{1}{12y^2-3}$$

when $y=1$

$$\frac{dy}{dx}=\frac{1}{12-3}=\frac{1}{9}$$ ①

$$i.e\ \frac{d}{dx}\ g^{-1}(2)\ =\frac{1}{9}$$

so

$f'(2)=\dfrac{2\times\frac{1}{9}-1}{4}$ $=\dfrac{\frac{2}{9}-1}{4}$ $=\dfrac{\frac{-7}{9}}{4}$ $=\dfrac{-7}{36}$	①